

η -Deuteron Scattering

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Abstract

Eta-deuteron scattering lengths are calculated. A summation of the multiple scattering series is carried out and the result is checked against more involved calculations. The necessity to go beyond the fixed nucleon approximation is emphasized. It is shown that a quasibound or virtual state in the η -deuteron system may occur within the range of η -nucleon scattering lengths suggested by other experiments.

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I. INTRODUCTION

Few-body interactions of the η meson may complement our knowledge on the η -nucleon interaction. Of related interest is the possibility of η -nuclear quasi-bound states. Such states have been predicted by Haider and Liu [1] and Li *et al.* [2], when it was realised that the η -nucleon interaction is attractive. In few-nucleon systems these states are expected to be narrow, and thus easier to detect. So far there has been no direct experimental verification of this hypothesis. On the other hand, Wilkin [3] has suggested that an indirect effect of such a state is seen in the rapid slope of the $pd \rightarrow \eta^3\text{He}$ amplitude detected just above the η production threshold [4]. Another indication of the strong three-body ηpp correlations follows from the recent measurement of $pp \rightarrow pp\eta$ cross sections in the threshold region [5].

The η -deuteron is the easiest few-body system to describe. In this letter a simple formula is given to provide the ηd scattering matrix at energies below the deuteron breakup. Detailed calculations are done for the scattering length $A_{\eta d}$, where the η -nucleon scattering length $a_{\eta N}$ is considered as an input. In the limiting case of fixed nucleons, this formula is found to be consistent with earlier calculations of Ref. [6]. However, our model includes corrections involving effects of the continuum in the ηpp system, which are found to be necessary.

For large values of $\text{Re } a_{\eta N}$ in the region of 0.7 to 1.0 fm, suggested by some models, we find the ηd system to be close to binding. In this region $A_{\eta d}$ becomes large and depends on details of the ηN interaction model. In particular one finds strong dependence on the way the ηN scattering matrix is extrapolated to the region below the threshold. If the actual $A_{\eta d}$ turns out to be sizable, it will be detected from the analysis of the final state interactions in the $pd \rightarrow pd\eta$ scattering experiment performed at Celsus recently [7].

II. A FORMULA FOR THE η -DEUTERON SCATTERING MATRIX

The purpose of this section is to derive a simple formula that relates the low-energy meson-deuteron scattering amplitude to the meson-nucleon scattering length. The former is

found by summation of a multiple scattering series and is expressed in terms of a few basic multiple scattering integrals. In order to motivate the method we recall a simple formula for the scattering length of a meson on a pair of fixed nucleons [8,9]

$$A_{\eta d} = \frac{2a_{\eta N}\xi}{1 - \frac{a_{\eta N}}{R_d}\xi}, \quad (1)$$

where $a_{\eta N}$ is the meson-nucleon scattering length and R_d is the nucleon-nucleon distance. Eq. (1) is obtained in a simple way by setting a boundary condition for the meson wave function ψ at each scatterer $\psi'/\psi = 1/a_{\eta N}$. In the simplest version of genuinely fixed scatterers $\xi = 1$, but a simple correction $\xi = m_{\eta d}/\mu_{\eta N}$ is easy to implement. Here, the meson-deuteron reduced mass $m_{\eta d}$ corrects for the meson propagator ($1/R_d$), which has to be referred to the NN centre of mass system. The reduced meson-nucleon mass $\mu_{\eta N}$ is necessary to relate the meson-nucleon scattering lengths to the meson-nucleon potentials.

Already at this stage Eq. (1) is a fair representation of $A_{\eta d}$ for $\text{Re } a_{\eta N}$ of about 0.3 fm or less. In principle it handles also situations of $A_{\eta d} \rightarrow \infty$, i.e. the cases of meson-deuteron bound or virtual states close to threshold. The latter may occur already at $\text{Re } a_{\eta N} \approx 1$ fm, which is close to the range of the η -nucleon scattering lengths allowed by some models [10]. However, for such large $\text{Re } a_{\eta N}$ Eq. (1) becomes rather inaccurate. In the rest of this section we find necessary corrections, determine the virtual or quasibound state singularities and discuss other related calculations of $A_{\eta d}$.

Let us begin with a multiple scattering expansion that follows from the three-body Faddeev equations for a meson interacting with a pair of nucleons labeled 1,2. For the situation of meson-deuteron scattering below the deuteron breakup, the series for the η -deuteron T -matrix is:

$$\begin{aligned} T_{\eta d} = & t_1 + t_2 + t_1 G_0 t_2 + t_2 G_0 t_1 + t_2 G_0 t_1 G_0 t_2 + t_1 G_0 t_2 G_0 t_1 \\ & + (t_1 + t_2) G_{NN} (t_1 + t_2) + (t_1 + t_2) G_{NN} (t_1 + t_2) G_{NN} (t_1 + t_2) + \dots, \end{aligned} \quad (2)$$

where t_i is a meson-nucleon scattering matrix, G_0 is the free three-body propagator and $G_{NN} = G_0 T_{NN} G_0$ is that part of the three-body propagator which contains the nucleon-

nucleon scattering matrix T_{NN} . This expansion is performed in momentum space and appropriate integrations over the intermediate momenta are understood. The detailed notation and normalisation will be given later. Now the partial summation of the series for the scattering amplitude is performed. The latter is determined by an average

$$\langle \phi_d \psi_\eta | T_{\eta d}(E) | \phi_d \psi_\eta \rangle \equiv \langle T_{\eta d}(E) \rangle, \quad (3)$$

where ϕ_d is the deuteron and ψ_η is the free-meson s -wave function in the relative meson-deuteron momentum variables. We define the ηd scattering length as

$$A_{\eta d} = -(2\pi)^2 m_{\eta d} \langle T_{\eta d}(0) \rangle. \quad (4)$$

A partial summation of the series (2) for $\langle T \rangle$ is obtained by

$$\langle T_{\eta d}^1 \rangle = \frac{\langle T_{\eta d}^0 \rangle}{1 - \Omega_1 - \Sigma_1} \quad (5)$$

where $\langle T_{\eta d}^0 \rangle = \langle t_1 + t_2 \rangle$ is the impulse approximation and

$$\Omega_1 = \frac{\langle t_1 G_0 t_2 + t_2 G_0 t_1 \rangle}{\langle T_{\eta d}^0 \rangle} \quad (6)$$

$$\Sigma_1 = \frac{\langle T_{\eta d}^0 G_{NN} T_{\eta d}^0 \rangle}{\langle T_{\eta d}^0 \rangle}. \quad (7)$$

This partial sum is already equivalent to formula (1) as it contains terms of the order $\lambda = \langle t \rangle / R_d$ in the denominator. The expansion parameter λ does not need to be small to guarantee the success of Eq. (5), which works even for $|\lambda| > 1$, when the multiple scattering is divergent. In the ηd case $|\lambda|$ falls into the 0.1–0.3 range. Corrections for higher orders of λ in the denominator of Eq. (5) may be obtained by comparing higher orders in Eq. (2) with a series expansion of Eq. (5) with respect to Σ_1 and Ω_1 . In this way the next approximation is obtained

$$\langle T_{\eta d}^2 \rangle = \frac{\langle T_{\eta d}^0 \rangle}{1 - \Omega_1 - \Sigma_1 - [\Omega_2 - (\Omega_1)^2] - [\Sigma_2 - (\Sigma_1)^2] - [\Delta_2 - \Omega_1 \Sigma_1]} \quad (8)$$

where

$$\Omega_2 = \frac{\langle t_1 G_0 t_2 G_0 t_1 + t_2 G_0 t_1 G_0 t_2 \rangle}{\langle T_{\eta d}^0 \rangle}, \quad (9)$$

$$\Sigma_2 = \frac{\langle T_{\eta d}^0 G_{NN} T_{\eta d}^0 G_{NN} T_{\eta d}^0 \rangle}{\langle T_{\eta d}^0 \rangle}, \quad (10)$$

and

$$\Delta_2 = 2 \frac{\langle T_{\eta d}^0 G_{NN} (t_1 G_0 t_2 + t_2 G_0 t_1) \rangle}{\langle T_{\eta d}^0 \rangle}. \quad (11)$$

This procedure may be continued into a systematic method to include higher powers of λ in the denominator. The main advantage is a strong cancellation in the $\Sigma_2 - (\Sigma_1)^2$ term and also higher order Σ_n terms, as was demonstrated in the optical model calculations of Ref. [11] for He nuclei. As we show numerically in the next section, also for the η -deuteron system there is a strong cancellation in the second (and indeed also in the higher orders) term of the partial sum (8). This causes the method to converge much more rapidly than the direct λ^n series.

Before going further we write down and discuss the basic quantities entering this formalism. Momentum variables are used everywhere. These are the momenta canonical to the Jacobi coordinates, \vec{q}_{NN} the relative NN momentum, \vec{p}_η the relative η - NN momentum, and the corresponding variables for the other possible pairs like $(\vec{q}_{\eta N}, \vec{p}_N)$. The normalisation is chosen so that a "volume" element is $d\vec{p}d\vec{q}$, the propagator $G_0 = (E - E_{NN}(q) - E_\eta(p))^{-1}$ and the scattering matrices are $t_{\eta N1}(q_{\eta N1}, q'_{\eta N1}, E - E(p_{N2}))\delta(\vec{p}_{N2} - \vec{p}'_{N2})$, i.e. they conserve the spectator momentum. The $t_{\eta N}$ are normalised in such a way that

$$t_{\eta N}(0, 0, 0) = -\frac{a_{\eta N}}{(2\pi)^2 \mu_{\eta N}} \quad (12)$$

with the standard convention $\text{Im } a_{\eta N} \geq 0$. Later, a separable form $t_{\eta N} = v_\eta(q_{\eta N})a_{\eta N}(E)v_\eta(q'_{\eta N})$ is used with a Yamaguchi formfactor $v_\eta = (1 + q_{\eta N}^2/\kappa_\eta^2)^{-1}$. The NN scattering matrix $T_{NN}(q_{NN}, q'_{NN}, E - E_\eta(p_\eta))$ is normalised with a different (standard) sign convention that requires

$$T_{NN}(0, 0, 0) = \frac{a_{NN}}{(2\pi)^2 \mu_{NN}}. \quad (13)$$

Also here a Yamaguchi separable form is used with $\kappa_{NN} = 1.41 \text{ fm}^{-1}$ and the strength fitted to reproduce the deuteron binding energy, the scattering length $a_{NN} = 5.405 \text{ fm}$.

Calculation of the multiple scattering integrals is straightforward although tedious and the formulae are lengthy. For simplicity we reproduce a few dominant quantities in the zero meson momentum and zero range η -nucleon force limit, although actual calculations are performed also taking into account a finite force range. Then the impulse approximation term becomes $\langle T_{\eta d}^0 \rangle = -2\bar{a}_{\eta N} / ((2\pi)^2 \mu_{\eta N})$, where

$$\bar{a}_{\eta N} = \int d\vec{p} a_{\eta N}(E - \frac{p^2}{2m_{N,\eta N}}) |\tilde{\phi}_d(q)|^2 \quad (14)$$

is the scattering matrix averaged over some energy region, generated by the recoil of the spectator nucleon. The range of the latter is given by the Fourier transform of the deuteron wave function $\tilde{\phi}$. In a more general nonzero meson momentum case the average is given by the momentum distribution of the ηN pair.

The quantity of interest is the scattering length at threshold $A_{\eta d}$. Hence, $E = -E_d$, where E_d is the deuteron binding energy, and the energies in Eq. (14) extend down to the subthreshold region. This means an extrapolation into the unphysical region by a few MeV. Therefore, a model is required for this extrapolation and some possibilities are discussed later. In general, due to the short range of ηN forces, the absence of nearby singularities or an ηN quasi-bound state, the energy dependence of $a_{\eta N}(E)$ in the narrow subthreshold region is apparently smooth. In all multiple scattering integrals of interest the average value \bar{a} is used. In this way the dominant term Σ_1 becomes

$$\Sigma_1 = -2\bar{a}_{\eta N} \int \frac{d\vec{p}}{(2\pi)^2 \mu_{\eta N}} T_{NN}(E - \frac{p^2}{2m_{\eta d}}) [F(p)]^2 \equiv 2\bar{a}_{\eta N} \sigma_1 \quad (15)$$

with

$$F(p) = \int d\vec{q} \frac{\tilde{\phi}_d(q) v_{NN}(\vec{q} - \frac{1}{2}\vec{p})}{E - E_{NN}(\vec{q} - \frac{1}{2}\vec{p}) - E_\eta(p)}, \quad (16)$$

where v_{NN} is the Yamaguchi formfactor for the NN separable potential. In the low energy region the T_{NN} matrix is dominated by the deuteron pole. For $E = -E_d$, the integration in

Eq. (15) extends from the pole down to negative energies. When T_{NN} is limited to the pole term, and the NN recoil energy E_{NN} is neglected in Eq. (16), expression (15) reduces to a simple form

$$\sigma_1 \approx \frac{m_{\eta d}}{\mu_{\eta N}} \int \int d\vec{r} d\vec{r}' \phi_d^2(r) \frac{1}{|\vec{r} - \vec{r}'|} \phi_d^2(r') \quad (17)$$

with a clear physical interpretation. The higher order terms for Σ_n have the same structure corresponding to $(n + 1)$ scatterings on the optical potential $V_{\eta d} = -2a_{\eta N}\phi_d^2(r)/(2\pi\mu_{\eta N})$ at zero incident energy.

Similarly to Refs. [11,12] it can be expected that the series for $T_{\eta d}^0, T_{\eta d}^1, T_{\eta d}^2, \dots$ converges so rapidly that $T_{\eta d}^2$ would be precise on the 1% level even in the case of a bound state at threshold. Indeed, the effectiveness of this expansion is confirmed in Table I for two particular values of the ηN scattering lengths and an ηN formfactor allowing comparison with the calculation of [6]. The latter one uses a rather involved set of integral equations for the η scattering on fixed nucleons corrected later for the effect of the deuteron wave functions, but no allowance is made for a free $NN\eta$ spectrum in the intermediate states. This assumption would correspond to our model with $\Omega_i = 0$. The agreement between these two calculations is rather good, with small differences probably due to two factors: first, Gaussian wave functions are used in Ref. [6], while ours come naturally in the Hulthen form; second, the T_{NN} used here contains more than the deuteron pole.

The effect of the free three-body spectrum in the intermediate states is still missing. To lowest order in $a_{\eta N}$ it is given by the Ω_1 term of Eq. (6). Within the average $\bar{a}_{\eta N}$ approximation it becomes

$$\Omega_1 = \bar{a}_{\eta N} \int \int \frac{d\vec{q}' d\vec{q}}{(2\pi)^2 \mu_{\eta N}} \frac{\tilde{\phi}_d(\vec{q}) \tilde{\phi}_d(-\vec{q}')}{E_{NN}(\frac{\vec{q}-\vec{q}'}{2}) + E_{\eta}(\vec{q} + \vec{q}') - E} \equiv \bar{a}_{\eta N} \omega_1. \quad (18)$$

This quantity is real below the deuteron breakup, which is the region of our interest. In a similar way one obtains higher order terms Ω_2 etc., which are also real in this region.

Now, the final formula to be used in the applications is presented as

$$A_{\eta d} = 2 \frac{m_{\eta d}}{\mu_{\eta N}} \frac{\bar{a}_{\eta N}}{1 - \bar{a}_{\eta N}(\omega_1 + 2\sigma_1) - \bar{a}_{\eta N}^2[(\omega_2 - \omega_1^2) + 4(\sigma_2 - \sigma_1^2) + 4(\delta_2 - \omega_1\sigma_1)]}, \quad (19)$$

where the second order terms are defined in analogy to Eqs. (15, 18), i.e. $\sigma_2 = \Sigma_2/(4\bar{a}_{\eta N}^2)$, $\omega_2 = \Omega_2/\bar{a}_{\eta N}^2$ and $\delta_2 = \Delta_2/(4\bar{a}_{\eta N}^2)$. The numerical factors 1, 2 and 4 in Eq. (19) arise from the number of independent collisions on the successive nucleons.

Numerical results for the scattering length are given in the next section. Before presenting these let us discuss the question of the unitarity of $\langle T_{\eta d}(E) \rangle$, when it is calculated for finite meson energies, but below the deuteron breakup threshold. Imaginary contributions to ω_i arise only above the deuteron breakup, but the absorptive parts of σ_i begin already at $E > -E_d$. These are generated by the deuteron pole in $T_{NN}(E - E_\eta(p))$ in the integral (15) for σ_1 and in similar formulas for the higher order σ_i . At $E = -E_d$, the pole term of $G_0 T_{NN} G_0$ contributes the $\phi_d(r)\phi_d(r')/|\vec{r} - \vec{r}'|$ term to expression (17) in the coordinate representation. For higher energies the meson propagator becomes $\exp(ip_{\eta d}|\vec{r} - \vec{r}'|)/|\vec{r} - \vec{r}'|$, where $p_{\eta d}$ is the incident meson momentum. Hence for small momenta we have $\text{Im } \sigma_1 \approx ip_{\eta d}m_{\eta d}/\mu_{\eta N}$. In addition the second term $\sigma_2 - \sigma_1^2$ generates no terms linear in $p_{\eta d}$ as may be seen from Eq. (17) and its second order analog. This allows us to present the low energy behaviour of $\langle T_{\eta d} \rangle$ as

$$\langle T_{\eta d}(E) \rangle (2\pi)^2 m_{\eta d} = -[\frac{1}{A_{\eta d}(E)} - ip_{\eta d}]^{-1} \quad (20)$$

as required by unitarity. It also permits the use of the series summation method in some energy region close to the threshold.

III. RESULTS

The formula for the meson-deuteron scattering length $A_{\eta d}$ expresses it in terms of an "effective η -nucleon scattering length" $\bar{a}_{\eta N}$, which is an average of the ηN scattering matrix extrapolated by a few MeV below the threshold. This is our input parameter. Inherently there is another parameter, the inverse range $\kappa_{\eta N}$ in the ηN form factor, included in the calculated quantities multiplying powers of the $\bar{a}_{\eta N}$ in the expansion (19) for the inverse $1/A_{\eta d}$. These follow from the three-body interactions, and they are given in Table II for

four representative values of $\kappa_{\eta N}$ from the literature [6,15,16], which presumably cover the whole range of allowed values. It follows from this table that the coefficients of the \bar{a}^2 order are very small. The second order terms $\sigma_2 - \sigma_1^2$ due to the intermediate NN interactions have been found to be small already in the optical model calculations of η He scattering [11]. Higher order terms are negligible as may be seen from the results of Table I.

As shown in Table I the static nucleon approximation is not adequate in the region of large $\text{Re } a_{\eta N}$ close to the critical values. The effect of the NN continuum is significant. Some care is needed with calculations of the corresponding scattering integrals in the three-body continuum ω_i . These converge slowly and depend strongly on the range of the ηN interaction. In particular, the zero range limit cannot be taken for ω_3 and higher orders. Provided an unphysical value $\kappa_{\eta N} = \infty$ is not used to describe the free-spectrum contributions, the small weight and the cancellations in $\omega_2 - \omega_1^2$ reduce effects of the continuum to the ω_1 term. A similar cancellation happens in the "mixed term" $\delta_2 - \omega_1 \sigma_1$ which, although small, dominates the second order term. All this allows the simple formula (19) to work very effectively even under the demanding condition of a nearby singularity.

In Table III one finds values of $A_{\eta d}$ for a number of ηN scattering lengths that follow from several analyses of the combined $\pi N, \eta N$ coupled channels and from η photoproduction data. The resulting $A_{\eta d}$ may vary by an order of magnitude, reflecting a nearby ηd quasibound state that may arise at threshold for $\text{Re } a_{\eta N}$ close to 0.8 fm. It is also clear that the results depend on the extrapolation of $a_{\eta N}(E)$ below the threshold. No detailed models are available in this region and two simple approaches have been attempted:

(1) a constant $a_{\eta N}$ and

(2) a typical low-energy form $a(E) = a_{\eta N}/(1 - iq_{\eta N}a_{\eta N})$ as required by unitarity.

In the latter case the calculations are done using an imaginary $q_{\eta N} = i 0.367 \text{ fm}^{-1}$. This value follows from the average value of the subthreshold energy argument $(-E_d - \frac{p^2}{2m_{N,\eta N}})$ involved in Eq. (14), (the recoil energy amounts to some 4 MeV). Such an extrapolation reduces the effective values of $\text{Re } \bar{a}_{\eta N}$ by 10 – 20% as compared to the threshold $\text{Re } a_{\eta N}$ values. The sensitivity on this extrapolation method is due to the nearby singularity. This

singularity may represent a quasi-bound ηd state, if $\text{Re } A_{\eta d} < 0$, or a quasi-virtual ηd state, if $\text{Re } A_{\eta d} > 0$.

A more systematic study of $A_{\eta d}$ is presented in Figs. 1 and 2, where plots of $\text{Im } a_{\eta d}$ and $\text{Re } a_{\eta d}$ are given for two typical values of $\text{Im } a_{\eta N}$ as functions $\text{Re } a_{\eta N}$. These are all calculated for $\kappa_\eta = 3.316 \text{ fm}^{-1}$ and for the two low energy presentations of $a(E)$. It is seen that the critical value of $\text{Re } a_{\eta N}$, when the virtual state is formed at the ηd threshold and crosses-over to the quasi-bound state is about 0.8 fm. That is well within the range of expected values. If this happens, one may observe very strong effects in the final state ηd interactions. These may be seen in $pd \rightarrow pd\eta$ scattering experiments, if relative ηd momenta as small as 10-60 MeV/c are measured.

IV. CONCLUSION

We have applied a multiple scattering series formalism developed earlier for the η -helium system to calculate the η -deuteron scattering length.

The conclusions are as follows:

- a) It is possible to sum effectively the multiple scattering series for the inverse meson-deuteron scattering matrix below the breakup threshold. The method has the advantage of being able to incorporate both the influence of NN scattering and the free three-body intermediate continuum in a relatively simple but numerically stable way without resorting to exact Faddeev equations.
- b) For small values of $\text{Re } a_{\eta N} \leq 0.3 \text{ fm}$ the fixed nucleons make a good approximation. For larger values of $\text{Re } a_{\eta N}$ it is necessary to take into account the intermediate three-body continuum states. The sensitivity to otherwise small effects is due to the proximity of an ηd quasibound state. For the same reason the ηd scattering length is fairly sensitive to the subthreshold extrapolation of the ηN scattering matrix.
- c) A virtual or quasi-bound ηd system is likely to be formed. For $\text{Re } a_{\eta N} \approx 0.8 \text{ fm}$ it occurs close to the threshold. Such a situation may be easily detected via final state interaction

studies.

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FIGURES

FIG. 1. The real and imaginary parts of the η -deuteron scattering length as a function of $\text{Re } a_{\eta N}$ for the constant extrapolation of the $a_{\eta N}$ below threshold and for $\kappa_{\eta N} = 3.316 \text{ fm}^{-1}$. The curves are given for $\text{Im } a = 0.2 \text{ fm}$ (solid) and for $\text{Im } a = 0.3 \text{ fm}$ (dashed).

FIG. 2. The real and imaginary parts of the η -deuteron scattering length as a function of $\text{Re } a_{\eta, N}$ for the energy-dependent extrapolation $a_{\eta N}(E) = a_{\eta N}/(1 - iq_{\eta N}a_{\eta N})$ below threshold and for $\kappa_{\eta N} = 3.316 \text{ fm}^{-1}$. The curves are given for $\text{Im } a = 0.2 \text{ fm}$ (solid) and for $\text{Im } a = 0.3 \text{ fm}$ (dashed).

TABLES

TABLE I. Convergence of the $A_{\eta d}$ expansion for two values of $\bar{a}_{\eta N} = a_{\eta N}$. Units are fm and

$\kappa_{\eta N} = 3.316\text{fm}^{-1}$.

$a_{\eta N}$	$0.27 + i0.22$	$0.55 + i0.30$	
$A_{\eta d}^0$	$0.66 + i0.54$	$1.35 + i 0.74$	$\sigma_i = 0 \quad \omega_i = 0$
$A_{\eta d}^1$	$0.66 + i0.83$	$1.56 + i 1.96$	$\sigma_1 \neq 0 \quad \omega_i = 0$
$A_{\eta d}^2$	$0.64 + i0.85$	$1.37 + i 2.14$	$\sigma_2 \neq 0 \quad \omega_i = 0$
$A_{\eta d}^3$	$0.64 + i0.85$	$1.37 + i 2.14$	$\sigma_3 \neq 0 \quad \omega_i = 0$
Ref. [6]	$0.65 + i0.85$	$1.38 + i 2.15$	
$A_{\eta d}^2$	$0.57 + i0.97$	$0.61 + i 2.73$	$\sigma_i \neq 0 \quad \omega_i \neq 0$

TABLE II. The calculated integral quantities in the multiple scattering sum for $A_{\eta d}$ as a function of $\kappa_{\eta N}$. Units are fm^{-1} for (σ_1, ω_1) and fm^{-2} for (σ_2, ω_2) .

$\kappa_{\eta} \text{ (fm}^{-1}\text{)}$	∞	7.617	3.316	2.357
σ_1	0.4440	0.4331	0.3966	0.3679
σ_2	0.2315	0.2245	0.1966	0.1588
ω_1	0.3118	0.2958	0.2546	0.2246
ω_2	0.2800	0.1870	0.1045	0.0723
δ_2	0.1759	0.1620	0.1249	0.1001
$\sigma_2 - \sigma_1^2$	0.0180	0.0075	0.0032	-0.0039
$\omega_2 - \omega_1^2$	0.1828	0.0874	0.0027	-0.0025
$\delta_2 - \omega_1\sigma_1$	0.0374	0.0338	0.0239	0.0174

TABLE III. Eta-deuteron scattering lengths in fm, $\kappa_{\eta N} = 3.316 fm^{-1}$.

	$a_{\eta N}$	$A_{\eta d}$	
		$a(E) = a_{\eta N}$	$a(E) = a_{\eta N}/(1 - iq_{\eta N}a_{\eta N})$
Bhalerao-Liu [15]	$0.27 + i 0.22$	$0.57 + i 0.97$	$0.64 + i 0.81$
"modified B-L" [15,10]	$0.44 + i 0.30$	$0.63 + i 1.93$	$1.01 + i 1.50$
Bennhold-Tanabe [16]	$0.25 + i 0.16$	$0.66 + i 0.71$	$0.66 + i 0.58$
"modified B-T" [16,10]	$0.46 + i 0.29$	$0.72 + i 2.04$	$1.11 + i 1.54$
Abaev-Nefkens [17]	$0.62 + i 0.30$	$0.36 + i 3.36$	$1.65 + i 2.41$
Wilkin [3]	$0.30 + i .30$	$0.39 + i 1.28$	$0.58 + i 1.11$
	$0.55 + i 0.30$	$0.61 + i 2.73$	$1.40 + i 1.98$
Arima [18]	$0.98 + i 0.37$	$-2.75 + i 2.77$	$-0.06 + i 6.20$
Sauerman [19]	$0.51 + i 0.21$	$1.48 + i 2.31$	$1.65 + i 1.39$
Batinic [10]	$0.888 + i 0.274$	$-2.90 + i 4.12$	$2.37 + i 5.79$
	$0.876 + i 0.274$	$-2.76 + i 4.24$	$2.42 + i 5.55$
Tiator [14]	$0.476 + i 0.279$	$0.81 + i 2.15$	$1.22 + i 1.56$
Krusche [13]	$0.430 + i 0.394$	$0.14 + i 1.91$	$0.65 + i 1.73$
	$0.579 + i 0.399$	$-0.13 + i 2.64$	$0.93 + i 2.41$
	$0.291 + i 0.360$	$0.17 + i 1.35$	$0.42 + i 1.25$



